Probabilistic approaches to discrete integrable systems

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The KdV equation and the Toda lattice are two central and widely-studied examples of classical integrable systems, and many of their variations have been introduced to the present. In particular, the box-ball system (BBS) is a basic example of a discrete integrable system, which has been revealed to be an ultra-discrete version of the KdV equation and the Toda lattice. The BBS has been studied from various viewpoints such as tropical geometry, combinatorics, and cellular-automaton. As a new perspective, research on probabilistic approaches to this system has been rapidly expanding in recent years, including the application of the Pitman transform, analysis of invariant measures and its generalized hydrodynamics. More recently, we find that the application of the Pitman transform and the study of invariant measures of i.i.d.-type also work in the same manner for the discrete KdV equation and the discrete Toda lattice [1, 2]. Further research has begun on the relationship between the Yang-baxter maps and the existence of i.i.d.-type invariant measures for the discrete integrable systems [3]. In this talk, I will introduce these new research topics that have been spreading over the past several years from the basics.

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References

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